Calculating Exponential Growth

It’s All In the Numbers

by

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RESOURCES

Free, online presentations, downloadable activities in PDF format, and annotated slide sets for classroom use are available at www.bioedonline.org or www.k8science.org.

CONTENT ADVISORY

See the following resources for additional information about HIV/AIDS and advice for discussing HIV/AIDS with students.

- National Institute of Allergy and Infectious Diseases, National Institutes of Health (NIH), offers resources on understanding HIV/AIDS: niaid.nih.gov/topics/hiv/aids and aidsinfo.nih.gov.
- National Institute on Drug Abuse, NIH, offers facts about drug abuse and the link between it and HIV/AIDS: hiv.drugabuse.gov.
- The Centers for Disease Control and Prevention provides up-to-date information on HIV/AIDS prevention: cdc.gov/hiv/topics.
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SOURCE URLs

AMERICAN DENTAL EDUCATION ASSOCIATION
explorehealthcareers.org

BAYLOR COLLEGE OF MEDICINE
BIOED ONLINE TEACHER RESOURCES
bioedonline.org | k8science.org
BAYLOR-UT CENTER FOR AIDS RESEARCH
bcm.edu/cfar
MOLECULAR VIROLOGY AND MICROBIOLOGY
bcm.edu/molvir

DARTMOUTH COLLEGE
ELECTRON MICROSCOPE FACILITY
dartmouth.edu/~emlab/

THE HENRY J. KAISER FAMILY FOUNDATION
kff.org

JOURNAL OF NANOBiOTECHNOLOGY
jnannoteknology.com/content/3/1/6

NATIONAL INSTITUTES OF HEALTH
LIFEWORKS
science.education.nih.gov/lifeworks

NATIONAL CENTER FOR RESEARCH RESOURCES
ncrr.nih.gov
NATIONAL INSTITUTE OF ALLERGY AND INFECTIOUS DISEASES
www.niaid.nih.gov
aidsinfo.nih.gov
NATIONAL INSTITUTE ON DRUG ABUSE
hivdrugabuse.gov
NATIONAL LIBRARY OF MEDICINE
nml.nih.gov/hmd

SCIENCE EDUCATION PARTNERSHIP AWARD
ncrrsepa.org

SUMANIS, INC.
ANIMATED TUTORIALS: MICROBIOLOGY
http://sumanisinc.com/webcontent/animation.html

U.S. CENTERS FOR DISEASE CONTROL AND PREVENTION (CDC)
HIV/AIDS PREVENTION
cdc.gov/hiv/topics
PUBLIC HEALTH IMAGE LIBRARY
phil.cdc.gov

U.S. CENTRAL INTELLIGENCE AGENCY
THE WORLD FACTBOOK

WELLCOME IMAGES
images.wellcome.ac.uk

WHAT IS PUBLIC HEALTH
whatispublichealth.org

WIKIMEDIA COMMONS
commons.wikimedia.org
**INTRODUCTION**

Microbial Challenges

Infectious diseases have plagued humans throughout history. Sometimes, they even have shaped history. Ancient plagues, the Black Death of the Middle Ages, and the “Spanish flu” pandemic of 1918 are but a few examples.

Epidemics and pandemics always have had major social and economic impacts on affected populations, but in our current interconnected world, the outcomes can be truly global. Consider the SARS outbreak of early 2003. This epidemic demonstrated that new infectious diseases are just a plane trip away, as the disease was spread rapidly to Canada, the U.S. and Europe by air travelers. Even though the SARS outbreak was relatively short-lived and geographically contained, fear inspired by the epidemic led to travel restrictions and the closing of schools, stores, factories and airports. The economic loss to Asian countries was estimated at $18 billion.

The HIV/AIDS viral epidemic, particularly in Africa, illustrates the economic and social effects of a prolonged and widespread infection. The disproportionate loss of the most economically productive individuals within the population has reduced workforces and economic growth in many countries, especially those with high infection rates. This affects the health care, education, and political stability of these nations.

In the southern regions of Africa, where the infection rate is highest, life expectancy has plummeted in a single decade, from 62 years in 1990–95 to 48 years in 2000–05. By 2003, 12 million children under the age of 18 were orphaned by HIV/AIDS in this region.

Despite significant advances in infectious disease research and treatment, control and eradication of diseases are slowed by the following challenges.

- The emergence of new infectious diseases
- An increase in the incidence or geographical distribution of old infectious diseases
- The re-emergence of old infectious diseases
- The potential for intentional introduction of infectious agents by bioterrorists
- The increasing resistance of pathogens to current antimicrobial drugs
- Breakdowns in public health systems.

Baylor College of Medicine, Department of Molecular Virology and Microbiology, bcm.edu/molvir.

**USING COOPERATIVE GROUPS IN THE CLASSROOM**

Cooperative learning is a systematic way for students to work together in groups of two to four. It provides organized group interaction and enables students to share ideas and to learn from one another. Students in such an environment are more likely to take responsibility for their own learning.

Cooperative groups enable the teacher to conduct hands-on investigations with fewer materials.

Organization is essential for cooperative learning to occur in a hands-on science classroom. Materials must be managed, investigations conducted, results recorded, and clean-up directed and carried out. Each student must have a specific role, or chaos may result.

The Teaming Up! model* provides an efficient system for cooperative learning.

Four “jobs” entail specific duties. Students wear job badges that describe their duties. Tasks are rotated within each group for different activities so that each student has a chance to experience all roles. For groups with fewer than four students, job assignments can be combined.

Once a model for learning is established in the classroom, students are able to conduct science activities in an organized and effective manner. Suggested job titles and duties follow.

**Principal Investigator**
- Reads the directions
- Asks the questions
- Checks the work

**Maintenance Director**
- Follows the safety rules
- Directs the cleanup
- Asks others to help

**Reporter**
- Records observations and results
- Explains the results
- Tells the teacher when the group is finished

**Materials Manager**
- Picks up the materials
- Uses the equipment
- Returns the materials

Acquired Immunodeficiency Syndrome (AIDS) is not a disease like the measles or flu, and there is no cure. It is the result of a long-term viral infection. A person with AIDS no longer has natural body protections against many diseases that circulate through the human population. People usually don’t die directly from HIV infection; rather, AIDS patients tend to suffer from chronic illnesses that accumulate one after another. Invading diseases gang up to waste away their bodies and cause great suffering until they no longer can survive.

Once a person has AIDS, treatment options are mostly reactive. If a person with AIDS has pneumonia or cancer, doctors employ pneumonia or cancer treatments. Often, AIDS patients have multiple illnesses, challenging doctors to find treatments that are effective and compatible. Regardless, over time the battle will be lost.

Because there is no cure or vaccine, worldwide efforts are focused on preventing AIDS from spreading from one person to the next. As noted earlier, AIDS results from infection by the Human Immunodeficiency Virus (HIV), an almost unimaginably small particle of genetic material more than 800 times smaller across than a human hair. HIV is passed from human to human only through body fluid transfer. Blood transfusions, breastfeeding, and sharing of needles among drug users are common routes of transfer of HIV virus particles.

HIV/AIDS: A NUMBERS GAME
Once inside the bloodstream, the virus particle attaches itself to cells that have a particular kind of molecule, called CD4, on their surface. T cells, the white blood cells responsible for directing the body’s defense against invaders, have CD4 receptor molecules. In fact, T cells also are referred to as CD4+ cells.

After attaching, the HIV virus particle injects its contents into the cell. The viral material may lay dormant for years but, eventually, it begins to multiply. Actually, the host cell does the multiplying. The particle simply provides the cell with a genetic "how-to" manual for creating copies of the virus.

Each new virus particle triggers the formation of more particles. Their numbers grow until millions of HIV particles are released into the bloodstream to interact with (infect) more CD4+ cells. Once infected, CD4+ cells are less able to defend the body against disease; sometimes, they are simply overwhelmed and die. As the immune system gradually fails, the disease known as AIDS results.

TREATMENT
Anti-HIV treatments usually rely on a combination of three different medications that target the HIV virus itself. Because HIV is capable of rapid genetic change (mutations), it can become resistant to the treatment drugs if medications are not taken on schedule as prescribed. HIV also is difficult to treat because its genetic material becomes incorporated into the DNA of cells within the human immune system. Once inside the nucleus of a CD4+ cell, for example, HIV can remain inactive and unaffected by drugs for years. HIV’s ability to “hide” within cells makes it impossible to eliminate completely. If treatment is stopped or disrupted for any reason, HIV is able to emerge from hiding and multiply within the body again.
Overview

Students will learn how to calculate exponential growth and, by extension, how to calculate the spread of HIV/AIDS. (See Answer Sheet on page 5.)

Calculating Exponential Growth

Under favorable conditions and with sufficient time and resources, populations of all organisms, including infectious agents like viruses, have the potential to increase dramatically over time. Even slow-growing organisms can reach astounding population sizes if reproduction is unchecked. Charles Darwin used elephants, which breed very slowly, as a hypothetical example. Beginning with two elephants, which generally produce only six offspring during a reproductive span of 60 years, an elephant population would number only 54 individuals after 200 years. However, after 1,000 years, the population would have grown to 86,000,000 elephants!

Now consider another example, in which a parent cell divides into two daughter cells every 10 minutes. After 10 minutes, there would be two cells; after 20 minutes, four cells; after 30 minutes, eight cells, and so on. After three hours, there would be close to one million cells. When quantity increases by a fixed percentage at regular time intervals, we have what is referred to as exponential growth. On a graph, exponential growth is represented by an upward curve, not a straight line. In addition to the example of cell division, exponential growth can be observed in the accumulation of compound interest, and in the increasing levels of CO\textsubscript{2} in the atmosphere. Untreated, HIV also is capable of exponential growth once it begins to replicate and spread within the human body.

MATERIALS
Teacher (see Setup)
- LCD or document projector, “smart-board” or overhead projector
- Slides or transparencies of student sheet
Per Student Group
- Calculator or computer access
- Spreadsheet software, if using a computer
- Copies of student sheet, “Dollars or Cents”

SETUP
If not using a document projector, prepare a slide or transparency of the spreadsheets. Also prepare slides or transparencies of the salary graph to show the difference between linear and exponential growth. Have students conduct this activity in groups of 2–4.

PROCEDURE
1. Lead a class discussion about the meaning of exponential growth, as it relates to HIV. Due to exponential growth, the greater the number of HIV particles present, the faster they will increase in number. Use the following example.

   If an HIV particle reproduces itself every minute, at the end of one minute, there will be two particles. After two minutes, there will be four particles; and after 10 minutes, the number will have grown to 1,024. In 20 minutes, Continued
there will be more than one million particles, and after 30 minutes, the population will have increased to more than one billion. This is “exponential” growth.

2. Tell students that there are many examples of exponential growth. Pose the following scenario to the class.

Imagine you have applied for a job. Your future employer offers a temporary position lasting just 30 days. Then, something amazing happens: you’re asked to decide if you’d rather be paid in dollars or pennies.

If you choose to be paid in dollars, you will earn $1,000 on the first day of work, $2,000 on your second day, $3,000 on the third, and so on. For each of your 30 days of employment, your salary will be increased by $1,000.

If you choose to be paid in pennies, you will earn one cent on the first day of work, two cents on your second day, four cents on the third day, and so on. Each day, your will salary will be exactly double the salary you earned the day before. Which payment plan will you select?

3. Give each student group the “Dollars or Cents” page, which includes the challenge just described. Allow time for students to discuss the options and select one of the job’s two possible “pay schedules.” Have students calculate their daily salaries, total income earned so far at the end of each day, and the amount of money they will earn for the full 30-day period.

4. Compare the final balances accrued by each salary schedule. If required for clarification, share the following information with students (also see the answer sheet at the end of this activity).

Being paid in dollars certainly seems like the smart choice. In just five days, you will earn $15,000. By the end of the next five days, your salary will reach $55,000. Adding $1,000 to your salary each day quickly builds up to a 30-day grand total of $465,000! Not bad for a temporary job.

On the other hand, it takes a lot of discipline (and quick calculations!) to choose to be paid in pennies. Initially, the pay will be dismal. By day 10, you will have only earned a total of only $10.23. It takes three weeks before your salary begins to pick up. On day 20, you will have earned $10,485.75. And from that point on, salary growth becomes spectacular. Just five days later, your salary will pass $335,000. By day 30, you will have earned $10,737,417.61!

5. Revisit your previous discussion of HIV replication. Ask students to explain how the salary analogy applies to virus multiplication within cells in the body. Or, ask each group of students to summarize what they learned about exponential growth by writing a paragraph in their science notebooks or as a homework assignment.
Imagine you have applied for a job. Your future employer offers a temporary position lasting just 30 days. Then, something amazing happens. You’re asked to decide if you’d rather be paid in dollars or pennies.

If you choose to be paid in dollars, you will earn $1,000 on the first day of work, $2,000 on your second day, $3,000 on the third, and so on. For each of your 30 days of employment, your salary will be increased by $1,000.

If you choose to be paid in pennies, you will earn one cent on the first day of work, two cents on your second day, four cents on the third day, and so on. Each day, your salary will be exactly double the salary you earned the day before. Which payment plan will you select?

1. Choose how you wish to be paid.
2. If you pick dollars per day, complete the chart on the left, beginning on Day 4. Add $1,000.00 to your salary each day until the column is completed.

   If you wish to be paid in pennies per day, complete the chart on the right, beginning on Day 4. Double your salary each day until the column is completed.
3. In the “Balance” column of your chart, add each day’s salary with the total from the day before and record each new balance through Day 30 to find how much you will earn in total.
4. Compare your salary balance on Day 30 with the balance of someone who picked the other pay schedule. Which of you will make more money? Why?
### Calculating Exponential Growth

#### The Science of HIV/AIDS: The Virus, the Epidemic and the World

**Micrococcus leuteus** bacteria. CD C\9761 Betsy Crane, Janice H. Carr.

### Answer Key

#### DAY 1
- **PAYMENT:** Dollars Per Day: $1,000.00
- **BALANCE:** $1,000.00

#### DAY 2
- **PAYMENT:** $2,000.00
- **PAYMENT:** Dollars Per Day: $3,000.00
- **BALANCE:** $3,000.00

#### DAY 3
- **PAYMENT:** $4,000.00
- **PAYMENT:** Dollars Per Day: $10,000.00
- **BALANCE:** $10,000.00

#### DAY 4
- **PAYMENT:** $5,000.00
- **PAYMENT:** Dollars Per Day: $15,000.00
- **BALANCE:** $15,000.00

#### DAY 5
- **PAYMENT:** $6,000.00
- **PAYMENT:** Dollars Per Day: $21,000.00
- **BALANCE:** $21,000.00

#### DAY 6
- **PAYMENT:** $7,000.00
- **PAYMENT:** Dollars Per Day: $28,000.00
- **BALANCE:** $28,000.00

#### DAY 7
- **PAYMENT:** $8,000.00
- **PAYMENT:** Dollars Per Day: $36,000.00
- **BALANCE:** $36,000.00

#### DAY 8
- **PAYMENT:** $9,000.00
- **PAYMENT:** Dollars Per Day: $45,000.00
- **BALANCE:** $45,000.00

#### DAY 9
- **PAYMENT:** $10,000.00
- **PAYMENT:** Dollars Per Day: $55,000.00
- **BALANCE:** $55,000.00

#### DAY 10
- **PAYMENT:** $11,000.00
- **PAYMENT:** Dollars Per Day: $66,000.00
- **BALANCE:** $66,000.00

#### DAY 11
- **PAYMENT:** $12,000.00
- **PAYMENT:** Dollars Per Day: $78,000.00
- **BALANCE:** $78,000.00

#### DAY 12
- **PAYMENT:** $13,000.00
- **PAYMENT:** Dollars Per Day: $91,000.00
- **BALANCE:** $91,000.00

#### DAY 13
- **PAYMENT:** $14,000.00
- **PAYMENT:** Dollars Per Day: $105,000.00
- **BALANCE:** $105,000.00

#### DAY 14
- **PAYMENT:** $15,000.00
- **PAYMENT:** Dollars Per Day: $120,000.00
- **BALANCE:** $120,000.00

#### DAY 15
- **PAYMENT:** $16,000.00
- **PAYMENT:** Dollars Per Day: $136,000.00
- **BALANCE:** $136,000.00

#### DAY 16
- **PAYMENT:** $17,000.00
- **PAYMENT:** Dollars Per Day: $153,000.00
- **BALANCE:** $153,000.00

#### DAY 17
- **PAYMENT:** $18,000.00
- **PAYMENT:** Dollars Per Day: $171,000.00
- **BALANCE:** $171,000.00

#### DAY 18
- **PAYMENT:** $19,000.00
- **PAYMENT:** Dollars Per Day: $190,000.00
- **BALANCE:** $190,000.00

#### DAY 19
- **PAYMENT:** $20,000.00
- **PAYMENT:** Dollars Per Day: $210,000.00
- **BALANCE:** $210,000.00

#### DAY 20
- **PAYMENT:** $21,000.00
- **PAYMENT:** Dollars Per Day: $231,000.00
- **BALANCE:** $231,000.00

#### DAY 21
- **PAYMENT:** $22,000.00
- **PAYMENT:** Dollars Per Day: $253,000.00
- **BALANCE:** $253,000.00

#### DAY 22
- **PAYMENT:** $23,000.00
- **PAYMENT:** Dollars Per Day: $276,000.00
- **BALANCE:** $276,000.00

#### DAY 23
- **PAYMENT:** $24,000.00
- **PAYMENT:** Dollars Per Day: $300,000.00
- **BALANCE:** $300,000.00

#### DAY 24
- **PAYMENT:** $25,000.00
- **PAYMENT:** Dollars Per Day: $325,000.00
- **BALANCE:** $325,000.00

#### DAY 25
- **PAYMENT:** $26,000.00
- **PAYMENT:** Dollars Per Day: $351,000.00
- **BALANCE:** $351,000.00

#### DAY 26
- **PAYMENT:** $27,000.00
- **PAYMENT:** Dollars Per Day: $378,000.00
- **BALANCE:** $378,000.00

#### DAY 27
- **PAYMENT:** $28,000.00
- **PAYMENT:** Dollars Per Day: $406,000.00
- **BALANCE:** $406,000.00

#### DAY 28
- **PAYMENT:** $29,000.00
- **PAYMENT:** Dollars Per Day: $435,000.00
- **BALANCE:** $435,000.00

#### DAY 29
- **PAYMENT:** $30,000.00
- **PAYMENT:** Dollars Per Day: $465,000.00
- **BALANCE:** $465,000.00

#### DAY 30

**Payment:** Pennies Per Day
- **PAYMENT:** $0.01
- **PAYMENT:** Dollars Per Day: $0.02
- **BALANCE:** $0.02

#### DAY 31
- **PAYMENT:** $0.04
- **PAYMENT:** Dollars Per Day: $0.08
- **BALANCE:** $0.08

#### DAY 32
- **PAYMENT:** $0.16
- **PAYMENT:** Dollars Per Day: $0.32
- **BALANCE:** $0.32

#### DAY 33
- **PAYMENT:** $0.64
- **PAYMENT:** Dollars Per Day: $1.28
- **BALANCE:** $1.28

#### DAY 34
- **PAYMENT:** $2.56
- **PAYMENT:** Dollars Per Day: $5.12
- **BALANCE:** $5.12

#### DAY 35
- **PAYMENT:** $10.23
- **PAYMENT:** Dollars Per Day: $20.47
- **BALANCE:** $20.47

#### DAY 36
- **PAYMENT:** $51.19
- **PAYMENT:** Dollars Per Day: $102.38
- **BALANCE:** $102.38

#### DAY 37
- **PAYMENT:** $255.98
- **PAYMENT:** Dollars Per Day: $511.95
- **BALANCE:** $511.95

#### DAY 38
- **PAYMENT:** $1,023.81
- **PAYMENT:** Dollars Per Day: $2,047.62
- **BALANCE:** $2,047.62

#### DAY 39
- **PAYMENT:** $4,143.03
- **PAYMENT:** Dollars Per Day: $8,286.06
- **BALANCE:** $8,286.06

#### DAY 40
- **PAYMENT:** $16,772.15
- **PAYMENT:** Dollars Per Day: $33,544.31
- **BALANCE:** $33,544.31

#### DAY 41
- **PAYMENT:** $67,108.32
- **PAYMENT:** Dollars Per Day: $134,217.63
- **BALANCE:** $134,217.63

#### DAY 42
- **PAYMENT:** $268,434.40
- **PAYMENT:** Dollars Per Day: $536,870.80
- **BALANCE:** $536,870.80

#### DAY 43
- **PAYMENT:** $1,073,417.61
- **PAYMENT:** Dollars Per Day: $2,146,821.62
- **BALANCE:** $2,146,821.62